

算例 4-002

轴对称实体 – 厚墙圆筒

问题描述

本例为一个厚墙形成的圆筒，按 MacNeal and Harder 1985 所述，承受 1ksi 的内压。本例中分别考虑平面应变和平面应力情况。用 $5 \times 1 \times 1$ 网格模拟圆筒中 10 度对应的一部分，厚度 1 英寸，内径 3 英寸，外径 9 英寸。将筒内侧的径向位移和径向、切向、长向应力分量与手算结果进行了比较。MacNeal and Harder 1985 中公布了平面应变状态下的径向位移值。

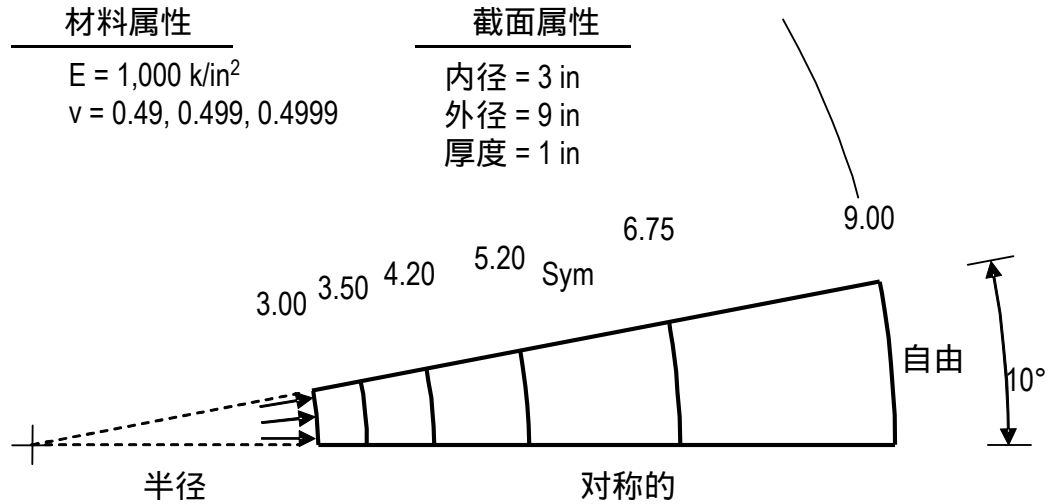
本例中使用四个不同泊松比和两个不同网格建模。下表列出了所使用的模型。

模型	泊松比	网格
A	0.3	$5 \times 1 \times 1$
B	0.49	$5 \times 1 \times 1$
C	0.499	$5 \times 1 \times 1$
D	0.4999	$5 \times 1 \times 1$
E	0.49	$20 \times 1 \times 1$
F	0.49	$160 \times 1 \times 1$

对所有模型使用不相容弯曲模式选项。

分析中 U_x 和 U_y 自由度为活动的。所有其他自由度都是不活动的。本例中对所有模型使用不相容弯曲模式选项。

几何特性、属性和荷载



校验的 SAP2000 的技术特色

- 使用轴对称实体单元进行的分析
- 轴对称面压力荷载

结果比较

使用从 Timoshenko 1956 中标题为 *Thermal Stresses in a Long, Hollow Cylinder* 的 44 章推导的公式来计算独立结果。MacNeal and Harder 1985 中给出了泊松比为 0.49、0.499 和 0.4999 时的径向位移独立结果。根据 Timoshenko 1956 中推导出的公式得到的手算径向位移与 MacNeal and Harder 1985 中给出的相同。

包括不相容模式选项

输出参数	模型、泊松比和网格	SAP2000	独立结果	差异百分比*
径向位移 in 内表面	A, $\nu = 0.3$, 5x1x1	0.004538	0.004582	-1%
	B, $\nu = 0.49$, 5x1x1	0.004971	0.005040	-1%
	C, $\nu = 0.499$, 5x1x1	0.004989	0.005060	-1%
	D, $\nu = 0.4999$, 5x1x1	0.004991	0.005062	-1%
	E, $\nu = 0.49$, 20x1x1	0.005035	0.005040	0%
	F, $\nu = 0.49$, 160x1x1	0.005040	0.005040	0%
径向应力 ksi 内表面	A, $\nu = 0.3$, 5x1x1	-0.834	-1.000	-17% (-13%)
	B, $\nu = 0.49$, 5x1x1	-0.833	-1.000	-17% (-13%)
	C, $\nu = 0.499$, 5x1x1	-0.833	-1.000	-17% (-13%)
	D, $\nu = 0.4999$, 5x1x1	-0.833	-1.000	-17% (-13%)
	E, $\nu = 0.49$, 20x1x1	-0.954	-1.000	-5% (-4%)
	F, $\nu = 0.49$, 160x1x1	-0.994	-1.000	-1% (-0%)
切向应力 ksi 内表面	A, $\nu = 0.3$, 5x1x1	1.293	1.250	3%
	B, $\nu = 0.49$, 5x1x1	1.364	1.250	9%
	C, $\nu = 0.499$, 5x1x1	1.369	1.250	10%
	D, $\nu = 0.4999$, 5x1x1	1.369	1.250	10%
	E, $\nu = 0.49$, 20x1x1	1.291	1.250	3%
	F, $\nu = 0.49$, 160x1x1	1.256	1.250	0%
纵向应力 ksi 内表面	A, $\nu = 0.3$, 5x1x1	0.138	0.075	84% (5%)
	B, $\nu = 0.49$, 5x1x1	0.260	0.123	111% (11%)
	C, $\nu = 0.499$, 5x1x1	0.267	0.125	114% (11%)
	D, $\nu = 0.4999$, 5x1x1	0.268	0.125	114% (11%)
	E, $\nu = 0.49$, 20x1x1	0.165	0.123	34% (3%)
	F, $\nu = 0.49$, 160x1x1	0.128	0.123	4% (0%)

* 括号中的差异百分比是相对于最大理论切向应力 1.250 ksi 的。

PROGRAM NAME: SAP2000
REVISION NO.: 2

计算模型文件： Example 4-002a, Example 4-002b, Example 4-002c
Example 4-002d, Example 4-002e, Example 4-002f

结论

与独立结果相比，当使用充分的剖分时，SAP2000 的结果可以接受。当网格细化时，SAP2000 解收敛于理论解。

手算过程

Plane Strain Solution

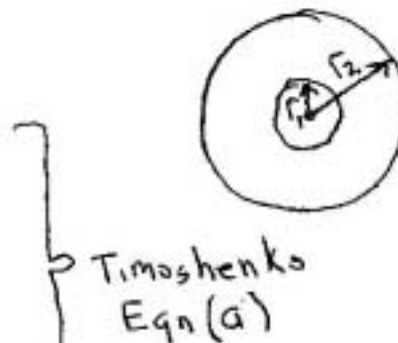
Reference: Timoshenko, 1956
Strength of Materials, Part II
Chapter VI, Section 44

U = radial disp)

$$\epsilon_r = \frac{du}{dr}$$

$$\epsilon_t = \frac{U}{r}$$

$$\epsilon_z = 0 \text{ (plane strain)}$$



$$\Delta = \text{Increase in unit volume}$$

$$\Delta = \epsilon_r + \epsilon_t + \epsilon_z = \epsilon_r + \epsilon_t \quad \left. \begin{array}{l} \text{Timoshenko} \\ \text{Eqn (c)} \end{array} \right\}$$

$$\sigma_r = \frac{E}{1+\nu} \left(\epsilon_r + \frac{\nu}{1-2\nu} \Delta \right)$$

$$\sigma_t = \frac{E}{1+\nu} \left(\epsilon_t + \frac{\nu}{1-2\nu} \Delta \right)$$

$$\sigma_z = \frac{E}{1+\nu} \left(\epsilon_z + \frac{\nu}{1-2\nu} \Delta \right) = \frac{E\nu\Delta}{(1+\nu)(1-2\nu)}$$

Timoshenko
Eqn (d)

The equation for radial displacement u is:

$$\frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} (ru) \right) = 0 \quad \text{Timoshenko Eqn (202)}$$

Integrate once to get

$$\frac{1}{r} \frac{d}{dr} (ru) = a$$

$$\frac{d}{dr} (ru) = ar$$

Integrate again to get

$$ru = ar^2 + b$$

$$\boxed{u = ar + \frac{b}{r}}$$

a & b are constants that will be derived later

$$\epsilon_r = \frac{du}{dr} = a - \frac{b}{r^2}$$

$$\epsilon_c = \frac{u}{r} = a + \frac{b}{r^2}$$

$$\begin{aligned}
 \sigma_r &= \frac{E}{1+\nu} \left(\epsilon_r + \frac{\nu}{1-2\nu} (\epsilon_r + \epsilon_t) \right) \\
 &= \frac{E}{1+\nu} \left(\epsilon_r \left(1 + \frac{\nu}{1-2\nu} \right) + \frac{\nu \epsilon_t}{1-2\nu} \right) \\
 &= \frac{E}{1+\nu} \left(\epsilon_r \left(\frac{1-\nu}{1-2\nu} \right) + \frac{\nu \epsilon_t}{1-2\nu} \right) \\
 &= \frac{E}{(1+\nu)(1-2\nu)} (\epsilon_r(1-\nu) + \nu \epsilon_t) \\
 &= \frac{E}{(1+\nu)(1-2\nu)} \left(\left(a - \frac{b}{r^2} \right) (1-\nu) + \nu \left(a + \frac{b}{r^2} \right) \right) \\
 &= \frac{E}{(1+\nu)(1-2\nu)} \left(a - a\nu + \frac{b}{r^2} + \frac{\nu b}{r^2} + \nu a + \frac{\nu b}{r^2} \right) \\
 &= \frac{E}{(1+\nu)(1-2\nu)} \left(a - \frac{b}{r^2} + \frac{2b\nu}{r^2} \right) \\
 \sigma_r &= \frac{E}{(1+\nu)(1-2\nu)} \left(a - (1-2\nu) \frac{b}{r^2} \right)
 \end{aligned}$$

At $r = r_2$, $\sigma_r = 0$, thus

$$a = (1-2\nu) \frac{b}{r_2^2}$$

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)} \left((1-2\nu) \frac{b}{r_2^2} - (1-2\nu) \frac{b}{r^2} \right)$$

$$\sigma_r = \frac{E}{1+\nu} \left(\frac{b}{r_2^2} - \frac{b}{r^2} \right)$$

At $r = r_1$, $\sigma_r = -P$, thus

$$-P = \frac{E}{1+\nu} \left(\frac{1}{r_2^2} - \frac{1}{r^2} \right) b$$

$$b = \frac{-P(1+\nu)}{E \left(\frac{1}{r_2^2} - \frac{1}{r^2} \right)}$$

$$\sigma_t = \frac{E}{1+\nu} \left(\epsilon_t + \frac{\nu}{1-\nu} (\epsilon_r + \epsilon_t) \right)$$

$$= \frac{E}{(1+\nu)(1-2\nu)} \left((1-\nu)\epsilon_t + \nu\epsilon_r \right)$$

$$= \frac{E}{(1+\nu)(1-2\nu)} \left((1-\nu) \left(a + \frac{b}{r^2} \right) + \nu \left(a - \frac{b}{r^2} \right) \right)$$

$$\sigma_t = \frac{E}{(1+\nu)(1-2\nu)} \left(a + \frac{b}{r^2} - a\nu - \frac{\nu b}{r^2} + a\nu - \frac{\nu b}{r^2} \right)$$

$$\begin{aligned}\sigma_t &= \frac{E}{(1+\nu)(1-2\nu)} \left(a + (1-2\nu) \frac{b}{r_2} \right) \\ &= \frac{E}{(1+\nu)(1-2\nu)} \left((1-2\nu) \frac{b}{r_2^2} + (1-2\nu) \frac{b}{r_2} \right)\end{aligned}$$

$$\boxed{\sigma_t = \frac{E}{1+\nu} \left(\frac{b}{r_2^2} + \frac{b}{r_2} \right)}$$

$$\begin{aligned}\sigma_z &= \frac{E\nu(\epsilon_r + \epsilon_t)}{(1+\nu)(1-2\nu)} \\ &= \frac{E\nu}{(1+\nu)(1-2\nu)} \left(a - \frac{b}{r_2} + a + \frac{b}{r_2} \right) \\ &= \frac{2E\nu a}{(1+\nu)(1-2\nu)} \\ &= \frac{2E\nu}{(1+\nu)(1-2\nu)} \times (1-2\nu) \frac{b}{r_2^2}\end{aligned}$$

$$\boxed{\sigma_z = \frac{2E\nu b}{(1+\nu)r_2^2}}$$

Summary of derived equations for
plane strain solution.

Constants

$$b = \frac{-P(1+\nu)}{E\left(\frac{1}{r_2^2} - \frac{1}{r^2}\right)}$$

$$a = (1-2\nu) \frac{b}{r_2^2}$$

Radial Displacement

$$U = ar + \frac{b}{r}$$

Stress

$$\sigma_r = \frac{E}{1+\nu} \left(\frac{b}{r_2^2} - \frac{b}{r^2} \right)$$

$$\sigma_t = \frac{E}{1+\nu} \left(\frac{b}{r_2^2} + \frac{b}{r^2} \right)$$

$$\sigma_z = \frac{2E\nu b}{(1+\nu) r_2^2}$$

Calculate constants for different poisson's ratios

V	P	E	r_2	r	b	a
0.3	1	1000	0	3	0.0131625	0.000065
0.49	1	1000	0	3	0.01508625	0.000003725
0.499	1	1000	0	3	0.015177375	3.7475E-07
0.4999	1	1000	0	3	0.015186488	3.74975E-08

Calculate radial displacement

V	Δr (in)
0.3	0.004583
0.49	0.005040
0.499	0.005060
0.4999	0.005062

Calculate stresses

V	σ_r (Ksi)	σ_e (Ksi)	σ_z (Ksi)
0.3	-1	1.25	0.075
0.49	-1	1.25	0.1225
0.499	-1	1.25	0.12475
0.4999	-1	1.25	0.124975